Winter 2015

Problem Set II: Due Wednesday, January 21, 2015

1.) Consult the book of your choice, to learn about the Kelvin-Helmholtz instability.

- a.) Calculate the growth rate for $\begin{array}{cccc}
 \mathcal{Q} & \rightarrow \mathcal{V}_{1} \\
 \mathcal{P}_{1} & \rightarrow \mathcal{V}_{2} \\
 \end{array}$ b.) $\begin{array}{ccccc}
 \mathcal{P}_{2} & \rightarrow \mathcal{V}_{1} \\
 \mathcal{P}_{2} & \rightarrow \mathcal{V}_{2} \\
 \mathcal{P}_{1} & \rightarrow \mathcal{V}_{2} \\
 \end{array}$ Consider both $\rho_{2} > \rho_{1}, \rho_{2} < \rho_{1}.$
- c.) Discuss your result.

d.) Now include surface tension in part b.).

- 2.) You toss a small rock in a pond.
 - a.) Describe quantitatively the long time behavior of the ripples, propagating away from the splash zone. What is the speed of the slowest ripple?
 - b.) If a large, asymmetric rock were tossed in, how long would it take for the radiated ripples to asymptote to what was described in a.)?

- 3.) Consider a system with $\partial \rho_0 / \partial z > 0$ and g downward.
 - a.) If $v, D \neq 0$, derive the condition for Rayleigh-Taylor marginality. Take the system in a box of height *h*, width *L*.
 - b.) If v > D, v < D, what are the forms of the growth rate?
- 4.) Consider 3.), now with a mean flow $\langle v_x(z) \rangle \hat{x}$.
 - a.) Derive the eigenmode equation for perturbations.
 - b.) Try to construct a criterion for characterizing this system with velocity shear and unstable stratification. [Hint: Explore the Richardson number.]. Map out regimes. [N.B. See Chandrasekhar or Drazin for Ri.]
- 5a.) Consider the system of 3.) with $v = D \neq 0$. Use mean field theory and linear response to estimate the flux of mass. The system in a box of height h, width L.
 - b.) How might you estimate amplitudes?